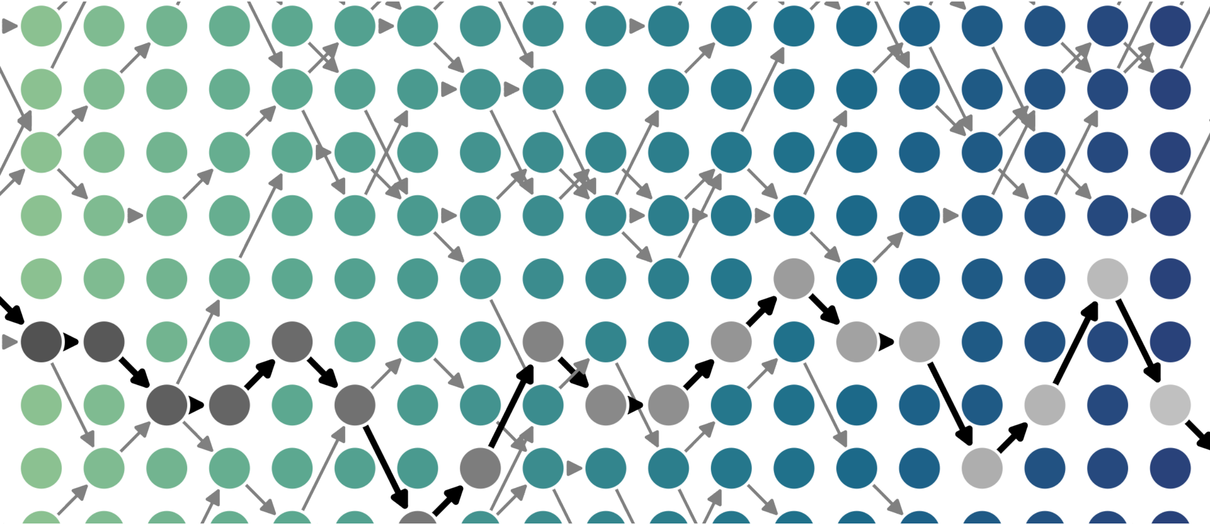
Dr. Timothy LaRock PhD

PRINCETON UNIVERSITY



Robustness of Directed Networks to Edge Set Removal

In collaboration with Jürgen Hackl (Princeton) and Yucong Li (Tongji)

NetSci 2026
June 4, 2026
Boston, MA

Network Robustness

Robustness of Complex Networks

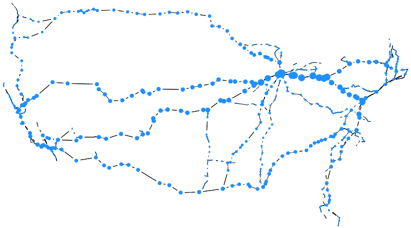
- Network connectivity can be **disrupted** by failure of nodes and edges
- Failures may be **random** (fallen tree branch) or **targeted** (natural disasters, adversarial attacks)
- Robustness asks: **how do repeated failures affect system connectivity and function**?

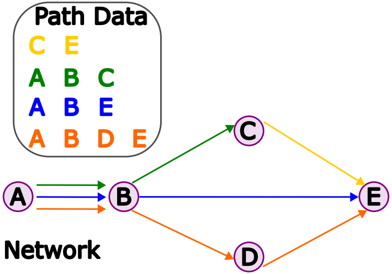
Path-based Perspective

- Many systems are characterized by **how people or cargo move through them**
- Navigation suggests importance of **directionality**, **paths**, and **dynamics** over the network

Change of Perspective

- How robust are networks with respect to the removal of (potentially correlated) **sets of edges**?





Inspiration: Shortest Path Percolation

Model

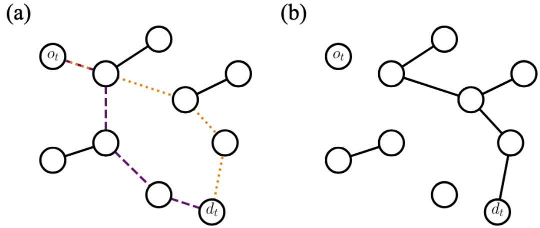
Choose origin ***o*** and destination ***d*** uniformly at random. If shortest path from ***o*** to ***d*** is shorter than a parameter C, remove one shortest path between them.

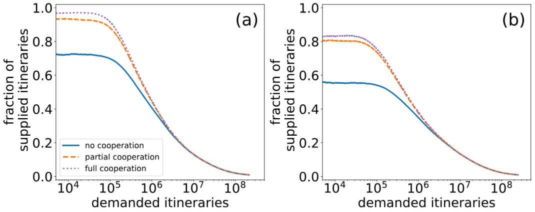
Results

- finite $C \longrightarrow$ percolation transition in same universality class as traditional percolation
- infinite $C \longrightarrow$ more abrupt transition
- Applied to line graph representation of a flight network $\longrightarrow$ found that cooperative multi-carrier itineraries increase network robustness

Generalized Edge Set Removal

- Observation: Shortest paths are just **ordered collections of edges**
- What if we **choose sets of edges in other ways**?





Kim & Radicchi, *PRL*, 2024; Kim *et al.*, *arXiv:2509.09142*, 2025; Kim *et al.*, *Nat Commun*, 2025. Parameswaran & Radicchi, Lightning Talk, NetSci 2026.

Framework: Generalized Edge Set Removal

Two components

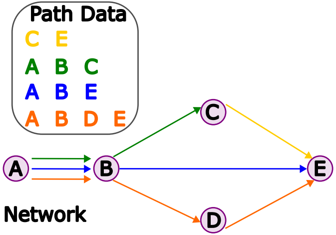
1. **universe** of possible edge sets
2. **probability function** or ranking to facilitate sampling

Example Edge Set Universes

- Uniform random edges
- Biased random edges (*e.g.*, weight or centrality)
- Observed path data
- Shortest paths
- Random walks

Example Ranking Functions

- Uniform random
- Number of edges
- Edge weights or centralities
- Statistical anomalies w.r.t. null model
- Machine learning derived path embeddings



Advantages

- Captures **wide variety of attack/failure modes** under single computational framework
- Allows for **"mixing and matching" of edge sets and ranking functions**, integrating **topology, dynamics, and inference/learning**

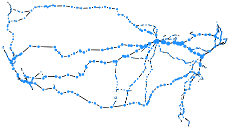
Preliminary Results

Datasets

Successful Wiki Games

- walks: 51307
- nodes: 4169
- edges: 60015
- largest WCC: 4169

AMTRAK®



Amtrak Routes Network

- walks: 490
- nodes: 650
- edges: 1624
- largest WCC: 636

Framework

Edge Set Universes

- **Observed walks** through the network
- **Shortest paths** through the network
- **Random walks** using distribution of **observed path lengths**
- **Random walks** using distribution of **shortest path lengths**

Rankings

- **Uniform** random
  - All edge sets
- **Length** biased
  - Both **static** and **dynamic** max edges
  - Only for observed walks

Results: Wikispeedia

Random walks disrupt wikispeedia

- Longer walks even more effective

Observed paths underperform

- However, sampling based on **dynamic length** performs much better than static length

Shortest paths

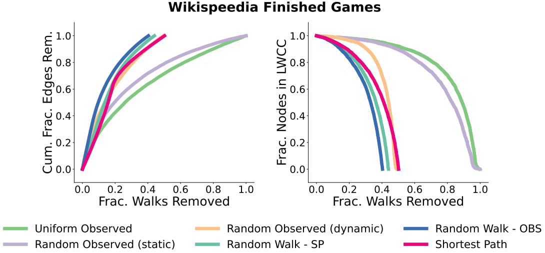
- Removing shortest paths falls between random walks and dynamic length observed

Caveat: underlying network structure

- Attacking the **effecitve network** constructed from games, not the hyperlink network
- Observed path coverage **too small to disrupt** the full hyperlink network!

Successful Wiki Games

- walks: 51307
- nodes: 4169
- edges: 60015
- largest WCC: 4169



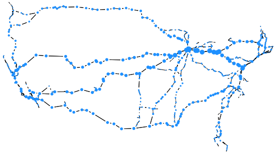
Results: Amtrak Routes

Shortest paths disrupt Amtrak network

- System designed for high efficiency and low redundancy

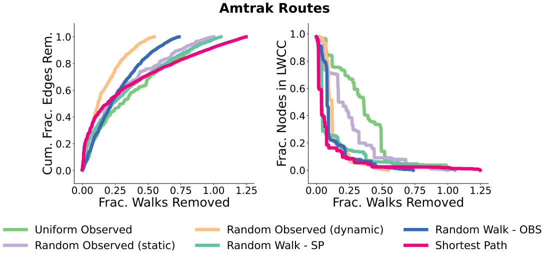
Shortest path lengths: fast to start, slow to finish

- Shortest path and random walk based on SP length are initially fast, but take longest to destroy the whole LWCC
- **Hypothesis**: as network disconnects, feasible paths get shorter (fewer edges), disruption slows



Amtrak Routes Network

- walks: 490
- nodes: 650
- edges: 1624
- largest WCC: 636



Results: Amtrak Routes

AMTRAK®

Shortest paths disrupt Amtrak network

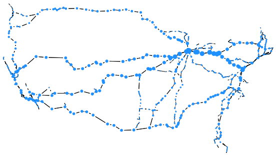
- System designed for high efficiency and low redundancy

Shortest path lengths: fast to start, slow to finish

- Shortest path and random walk based on SP length are initially fast, but take longest to destroy the whole LWCC
- **Hypothesis**: as network disconnects, feasible paths get shorter (fewer edges), disruption slows

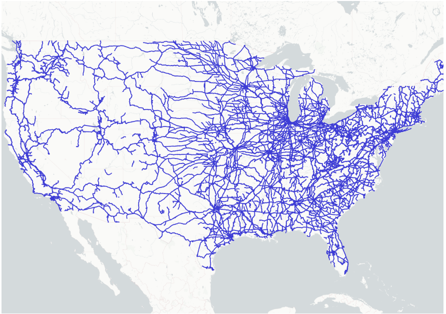
Caveats (redux)

- Data really represents paths of trains (and passengers)
- However, small piece of full north american rail network
- Not accounting for physical tracks, commuter and freight train interactions, etc.

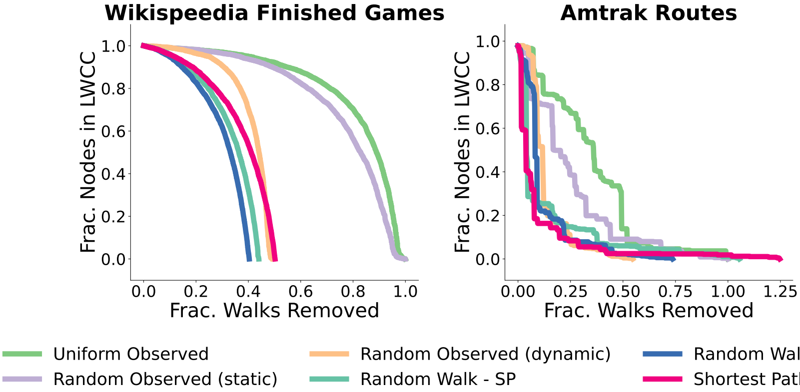


Amtrak Routes Network

- walks: 490
- nodes: 650
- edges: 1624
- largest WCC: 636



Direct Comparison



Random walks vs. shortest paths

- Random walks very disruptive in Wikipedia network
- Shortest paths very disruptive in Amtrak network

Observed path disruption

- Removing observed paths not a very effective strategy, except when dynamically biased towards long paths

Ongoing Work

Ranking Strategies

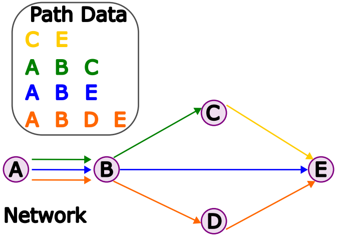
- Centrality measures
- Statistical importance (path anomaly detection)
- Machine learning methods

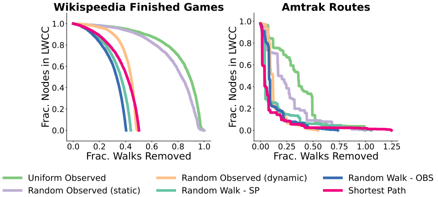
Dynamically informed connectivity measures

- Account for directionality, travel time, and spatial embeddedness
- Relate topological robustness with domain-specific dynamical disruptions
- More complex topological features

Network caveats

- Underlying network vs. effective network built from observed paths





Thank You!

Contact

Dr. Timothy LaRock
`larock@princeton.edu`
`https://tlarock.github.io`

PRINCETON ENGINEERING

Collaborators

Thanks to Yucong Li (Tongji University) and Professor Jürgen Hackl (Princeton).