CS 3000: Algorithms & Data — Summer 1 '20 — Tim LaRock

Homework 5
Due Monday June 1st at 11:59pm Boston time via Gradescope

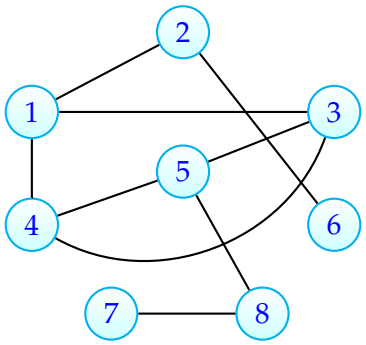
Name:
Collaborators:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This assignment is due Monday June 1st at 11:59pm Boston time via Gradescope. Make sure to submit something before the deadline.
- Solutions must be typeset in LaTeX. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.
- I encourage you to work with your classmates on the homework problems. *If you do collaborate, you must write all solutions by yourself, in your own words.* Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class, is strictly forbidden.

Problem 1. *Graph Representations and Exploration*

This problem tests your understanding of basic graph algorithms and concepts.

(a) Consider the following graph



(i) Construct the adjacency matrix of this graph. **Tip:** I included a snippet of code you can use to create a matrix in LaTeX.

Solution:

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

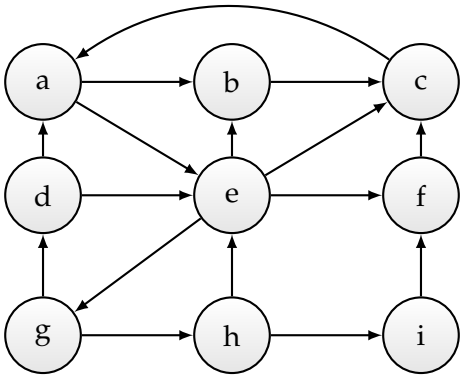
(ii) Construct the adjacency list of this graph.

Solution:

(iii) BFS this graph starting from the node 1. Always choose the lowest-numbered node next. Draw the BFS tree and label each node with its distance from 1.

Solution:

Problem 2. *DFS and Topological Ordering*



Consider running depth-first search on this graph starting from node *a*. When there are multiple choices for the next node to visit, go in alphabetical order.

(a) Label every edge as either tree, forward, backward, or cross.

Solution:

(b) Give the post-order numbers of all vertices

Solution:

a	b	c	d	e	f	g	h	i

(c) Is this graph a DAG? If so, give a topological ordering.

Solution:

Problem 3. *Graph Properties*

Consider an undirected graph $G = (V, E)$. The *degree* of a vertex v is the number of edges adjacent to v—that is, the number of edges of the form $(v, u) \in E$. Recall the standard notational convention that $n = |V|$ and $m = |E|$.

(a) Prove by induction that the sum of the degrees of the vertices is equal to $2m$.

Solution:

(b) Prove that there are an even number of vertices whose degree is odd.

Solution:

(c) Let $v \in V$ be some vertex whose degree is odd. Prove that there exists another vertex $u \in V$ such that u has odd degree and there is a path connecting v and u.

Solution: