Lecture 8: More Dynamic Programming

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bit.ly/cs3000syllabus

Business

Keep working on homework 2!

- Ask questions early if you are stuck!

Take home midterm 1 will be next Wednesday through Friday (more at the end)

Today

Brief correction on yesterday’s lecture

Dynamic Programming

- Subset Sum
- Edit Distance

Correct the record: 2 mistakes

$$f_n \begin{cases} 0 & if\ n = 0 \\ 1 & if\ n = 1 \\ f_{n-1} + f_{n-2} & otherwise \end{cases}$$

```
$Fib(n)$:
  If $n = 0$:
    return 0
  ElseIf $n = 1$:
    return 1
  Else:
    return $Fib(n-1)$ + $Fib(n-2)$
```

What does the recurrence relation $T(n)$ look like?

$$T(0) = 1, T(1) = 1$$

$$T(n) = T(n-1) + T(n-2) + 1$$

First, if we squint and assume $n \to \infty$ we might see

$$T(n) = T(n-1) + T(n-1) + 1$$

$$T(n) = 2T(n-1) + 1 \leq 2 \cdot 2^n$$

$$\leq O(2^{n+1})$$

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$$f_n \begin{cases} 0 & if\ n = 0 \\ 1 & if\ n = 1 \\ f_{n-1} + f_{n-2} & otherwise \end{cases}$$

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$$T(n) = 2T(n-1) + 1 \xcancel{\leq 2 \cdot 2^n}$$

$$\xcancel{\leq O(2^{n+1})}$$

This is wrong!
I was not careful
and made an
error!

Correct the record: 2 mistakes

$$f_n \begin{cases} 0 & if\ n = 0 \\ 1 & if\ n = 1 \\ f_{n-1} + f_{n-2} & otherwise \end{cases}$$

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$Fib(n)$:
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What does the recurrence relation $T(n)$ look like?

$$T(0) = 1, T(1) = 1$$

$$T(n) = T(n-1) + T(n-2) + 1$$

$$\begin{array}{l|l} T(2) = T(1) + T(0) + 1 = 3 & Fib(3) = Fib(2) + Fib(1) = 2 \\ T(3) = T(2) + T(1) + 1 = 5 & Fib(4) = Fib(3) + Fib(2) = 3 \\ T(4) = T(3) + T(2) + 1 = 9 & Fib(5) = Fib(4) + Fib(3) = 5 \end{array}$$

$$T(2) = 2Fib(2+1) - 1 = 3$$

$$T(3) = 2Fib(3+1) - 1 = 5$$

$$T(4) = 2Fib(4+1) - 1 = 9$$

$$T(n) = 2f_{n+1} - 1$$

Correct the record: 2 mistakes

$$f_n \begin{cases} 0 & if\ n = 0 \\ 1 & if\ n = 1 \\ f_{n-1} + f_{n-2} & otherwise \end{cases}$$

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$$T(2) = T(1) + T(0) + 1 = 3 \qquad Fib(3) = Fib(2) + Fib(1) = 2$$

$$T(3) = T(2) + T(1) + 1 = 5 \qquad Fib(4) = Fib(3) + Fib(2) = 3$$

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$$T(2) = 2Fib(2+1) - 1 = 3$$

$$T(3) = 2Fib(3+1) - 1 = 5$$

$$T(4) = 2Fib(4+1) - 1 = 9$$

$$T(n) = 2f_{n+1} - 1 \rightarrow 2T\xcancel{(n+1) \leq O}(2^{n+1})$$

This is wrong!

Correct the record: 2 mistakes

$T(n) = 2T(n-1)+1$

$$f_n \begin{cases} 0 & if\ n = 0 \\ 1 & if\ n = 1 \\ f_{n-1} + f_{n-2} & otherwise \end{cases}$$

What does the recurrence relation $T(n)$ look like?

$$T(0) = 1, T(1) = 1$$

$$T(n) = T(n-1) + T(n-2) + 1$$

$T(2) = T(1) + T(0) + 1 = 3$	$Fib(3) = Fib(2) + Fib(1) = 2$
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```
$Fib(n)$:
  If $n = 0$:
    return 0
  ElseIf $n = 1$:
    return 1
  Else:
    return $Fib(n-1)$ + $Fib(n-2)$
```

The important point is that just counting to f_n would be twice as fast!

$$T(2) = 2Fib(2+1) - 1 = 3$$

$$T(3) = 2Fib(3+1) - 1 = 5$$

$$T(4) = 2Fib(4+1) - 1 = 9$$

$$T(n) = 2f_{n+1} - 1$$

$2N-1$

$N = f_{n+1}$

Today: Dynamic Programming Subset Sum

Subset Sum

$$T(n) = 2T(n-1) + O(1) \leq O(2^n)$$

```
SubsetSum(X[1..n], i, T):
  If T = 0:
    return True
  ElseIf T < 0 or i = 0:
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  Else:
    with ← SubsetSum(X, i-1, T – X[i])
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We are given a set of n positive integers $X = \{x_{1,} x_{2,} \ldots, x_n\}$ and a target integer value T. We want to find a subset $\mathrm{Y} \subseteq X$ such that the sum of the elements $\sum_{x_i \in Y} x_i = T$.

Our problem: For a given T and X, does such a Y exist?

True or

False

Subset Sum

We are given a set of n positive integers $X = \{x_{1,} x_{2,} \ldots, x_n\}$ and a target integer value T. We want to find a subset $\mathrm{Y} \subseteq X$ such that the sum of the elements $\sum_{x_i \in Y} x_i = T$.

Our problem: For a given T and X, does such a Y exist?

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    return with OR wout
```

- We know our algorithm is correct, but it is very slow
- Let’s reformulate it with dynamic programming

Formulating Subset Sum for Dynamic Programming

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SubsetSum(X[1..n], i, T):
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Formulating Subset Sum for Dynamic Programming

What are our subproblems?

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What are our subproblems?

What data structure can we use for memoization?

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Which subproblems depend on each other, and what evaluation order does this imply?

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What are our subproblems?

Define a boolean function $SubSum(i, t)$ that returns $True$ if and only if there is a subset of $X[i..n]$ sums to t.

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Define a boolean function $SubSum(i, t)$ that returns $True$ if and only if there is a subset of $X[i..n]$ sums to t.

At an arbitrary iteration $1 \leq i \leq n + 1$ and $t \leq T$

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$$SubSum(i, t) = \left\{ \vphantom{\begin{matrix} a \\ b \\ c \end{matrix}} \right.$$

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$$SubSum(i,t) = \begin{cases} True & if\ t = 0 \\ & \\ & \end{cases}$$

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$$SubSum(i, t) = \begin{cases} True & if\ t = 0 \\ False & if\ i > n \\ & \\ & \end{cases}$$

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$$SubSum(i, t) = \begin{cases} True & if\ t = 0 \\ False & if\ i > n \\ SubSum(i + 1, t) & if\ t < X[i] \\ & \end{cases}$$

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Formulating Subset Sum for Dynamic Programming

What are our subproblems?

At an arbitrary iteration $1 \leq i \leq n+1$ and $t \leq T$

$$SS(i,t) = \begin{cases} \quad True & if\ t = 0 \\ \quad False & if\ i > n \\ SubSum(i+1,t) & if\ t < X[i] \\ SubSum(i+1,t)\ \vee SubSum(i+1,t-X[i]) & otherwise \end{cases}$$

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What data structure can we use for memoization?

Which subproblems depend on each other, and what evaluation order does this imply?

What are the space/time requirements?

What data structure can we use for memoization?

We can fill a 2 dimensional array with the following dimensions:

$$S[1..n+1, 0..T] = SubSum(i,t)$$

	$S(1{,}0)$	$S(1{,}1)$	$S(1{,}2)$	$S(1{,}3)$
	$S(2{,}0)$	$S(2{,}1)$	$S(2{,}2)$	$S(2{,}3)$
n	$S(3{,}0)$	$S(3{,}1)$	$S(3{,}2)$	$S(3{,}3)$
	$S(4{,}0)$	$S(4{,}1)$	$S(4{,}2)$	$S(4{,}3)$

T

Formulating Subset Sum for Dynamic Programming

What are our subproblems?

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n

$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
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We can fill a 2 dimensional array with the following dimensions: $S[1..n+1, 0..T] = SubSum(i,t)$

Which subproblems depend on each other, and what evaluation order does this imply?

$SubSum(i,t)$ can depend on $SubSum(i+1,t)$ and $SubSum(i+1,t-X[i])$. So we can start at the bottom of the table and work up.

n

$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
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T

What are the space/time requirements?

Formulating Subset Sum for Dynamic Programming

What are our subproblems?

At an arbitrary iteration $1 \leq i \leq n + 1$ and $t \leq T$

$$SS(i,t) = \begin{cases} True & if\ t = 0 \\ False & if\ i > n \\ SubSum(i+1,t) & if\ t < X[i] \\ SubSum(i+1,t)\ \vee SubSum(i+1, t - X[i]) & otherwise \end{cases}$$

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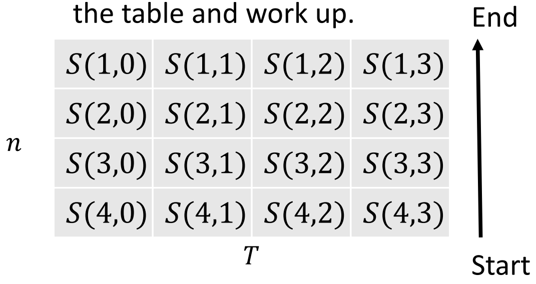

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What are the space/time requirements?

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T

Space requirement is $O(nT)$

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What are our subproblems?

At an arbitrary iteration $1 \leq i \leq n + 1$ and $t \leq T$

$$SS(i,t) = \begin{cases} True & if\ t = 0 \\ False & if\ i > n \\ SubSum(i+1,t) & if\ t < X[i] \\ SubSum(i+1,t) \vee SubSum(i+1,t-X[i]) & otherwise \end{cases}$$

What data structure can we use for memoization?

We can fill a 2 dimensional array with the following dimensions:
$S[1..n+1, 0..T] = SubSum(i,t)$

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What are the space/time requirements?

Space: $O(nT)$

```
SubsetSum(X[1..n], i, T):
  If T = 0:
    return True
  ElseIf T < 0 or i = 0:
    return False
  Else:
    with ← SubsetSum(X, i-1, T − X[i])
    wout ← SubsetSum(X, i-1, T)
    return with OR wout
```

What are the space/time requirements?

n

$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
$S(3,0)$	$S(3,1)$	$S(3,2)$	$S(3,3)$
$S(4,0)$	$S(4,1)$	$S(4,2)$	$S(4,3)$

T

Formulating Subset Sum for Dynamic Programming

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$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
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$S(3,0)$	$S(3,1)$	$S(3,2)$	$S(3,3)$
$S(4,0)$	$S(4,1)$	$S(4,2)$	$S(4,3)$

T

Using our evaluation order, we can fill the table in constant time per update, so the time complexity is also $O(nT)$

Formulating Subset Sum for Dynamic Programming

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FastSubsetSum(X[1..n], T):
  $S[n+1,0] \leftarrow True$
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    $S[n+1,t] \leftarrow False$
  for i$\leftarrow n$ down to 1:
    $S[i,0] \leftarrow True$
    for t$\leftarrow$1 to $X[i]-1$:
      $S[i,t] \leftarrow S[i+1,t]$
    for $t \leftarrow X[i]$ to $T$:
      $S[i,t] \leftarrow S[i+1,t] \vee S[i+1,t-X[i]]$
  return $S[1,T]$
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    for $t \leftarrow X[i]$ to $T$:
      $S[i,t] \leftarrow S[i+1,t] \vee S[i+1,t-X[i]]$
  return $S[1,T]$
```

Let’s see an example

Dynamic Programming Subset Sum Example

$i = X(i)$ $n = 3$

$X = [1,2,3], T = 3$

```
FastSubsetSum(X[1..n], T):
    $S[n+1,0] \leftarrow True$
    for $t \leftarrow 1$ to $T$:
        $S[n+1,t] \leftarrow False$
    for i$\leftarrow n$ down to 1:
        $S[i,0] \leftarrow True$
        for t$\leftarrow$1 to $X[i]-1$:
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        for $t \leftarrow X[i]$ to $T$:
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    return $S[1,T]$
```

$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
$S(3,0)$	$S(3,1)$	$S(3,2)$	$S(3,3)$
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Dynamic Programming Subset Sum Example

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    return $S[1,T]$
```

$$X \ = [1,2,3], T = 3$$

$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
$S(3,0)$	$S(3,1)$	$S(3,2)$	$S(3,3)$
T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
  $S[n+1,0] \leftarrow True$
  for $t \leftarrow 1$ to $T$:
    $S[n+1,t] \leftarrow False$
  for i$\leftarrow n$ down to 1:
    $S[i,0] \leftarrow True$
    for t$\leftarrow$1 to $X[i]-1$:
      $S[i,t] \leftarrow S[i+1,t]$
    for $t \leftarrow X[i]$ to $T$:
      $S[i,t] \leftarrow S[i+1,t] \vee S[i+1,t-X[i]]$
  return $S[1,T]$
```

Ignor

$X = [1,2,3], T = 3$

negative numbers but still fill the table

	$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
	$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
X[i] = 3	T	$S(3,1)$	$S(3,2)$	$S(3,3)$
	T	F	F	F

$t=1$

$t=2$

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
    $S[n+1,0] \leftarrow True$
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$X \ = [1,2,3], T = 3$

	$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
	$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
X[i] = 3	T	F	F	$S(3,3)$
	T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
  $S[n+1,0] \leftarrow True$
  for $t \leftarrow 1$ to $T$:
    $S[n+1,t] \leftarrow False$
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    $S[i,0] \leftarrow True$
    for t$\leftarrow$1 to $X[i]-1$:
      $S[i,t] \leftarrow S[i+1,t]$
    for $t \leftarrow X[i]$ to $T$:
      $S[i,t] \leftarrow S[i+1,t] \vee S[i+1,t-X[i]]$
  return $S[1,T]$
```

$Y = \{3\}$

$X = [1,2,3], T = 3$

	$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
	$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
X[i] = 3	T	F	F	T
	T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
  $S[n+1, 0] \leftarrow True$
  for $t \leftarrow 1$ to $T$:
    $S[n+1, t] \leftarrow False$
  for i$\leftarrow n$ down to 1:
    $S[i, 0] \leftarrow True$
    for t$\leftarrow 1$ to $X[i] - 1$:
      $S[i, t] \leftarrow S[i+1, t]$
    for $t \leftarrow X[i]$ to $T$:
      $S[i, t] \leftarrow S[i+1, t] \vee S[i+1, t - X[i]]$
  return $S[1, T]$
```

$$X \ = [1,2,3], T = 3$$

	$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
X[i] = 2	$S(2,0)$	$S(2,1)$	$S(2,2)$	$S(2,3)$
	T	F	F	T
	T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
  $S[n+1,0] \leftarrow True$
  for $t \leftarrow 1$ to $T$:
    $S[n+1,t] \leftarrow False$
  for i$\leftarrow n$ down to 1:
    $S[i,0] \leftarrow True$
    for t$\leftarrow$1 to $X[i]-1$:
      $S[i,t] \leftarrow S[i+1,t]$
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```

$X = [1,2,3], T = 3$

	$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
X[i] = 2	T	F	$S(2,2)$	$S(2,3)$
	T	F	F	T
	T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
  $S[n+1,0] \leftarrow True$
  for $t \leftarrow 1$ to $T$:
    $S[n+1,t] \leftarrow False$
  for i$\leftarrow n$ down to 1:
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      $S[i,t] \leftarrow S[i+1,t]$
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```

$$X = [1,2,3], T = 3$$

	$S(1,0)$	$S(1,1)$	$S(1,2)$	$S(1,3)$
X[i] = 2	T	F	T	$S(2,3)$
	T	F	F	T
	T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
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X[i] = 2	T	F	T	T
	T	F	F	T
	T	F	F	F

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        for $t \leftarrow X[i]$ to $T$:
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    return $S[1,T]$
```

$$X\ = [1,2,3], T = 3$$

X[i] = 1	T	$S(1,1)$	$S(1,2)$	$S(1,3)$
	T	F	T	T
	T	F	F	T
	T	F	F	F

Dynamic Programming Subset Sum Example

```
FastSubsetSum(X[1..n], T):
   $S[n+1,0] \leftarrow True$
   for $t \leftarrow 1$ to $T$:
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   for i$\leftarrow n$ down to 1:
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      for t$\leftarrow$1 to $X[i]-1$:
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X[i] = 1	T	$S(1,1)$	$S(1,2)$	$S(1,3)$
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	T	F	T	T
	T	F	F	T
	T	F	F	F

Since $S[1,3]$ checks $S[2,3]$ which is $True$, we know we have a solution!

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$X \ = [1,2,3], T = 3$

X[i] = 1	T	T	T	T
	T	F	T	T
	T	F	F	T
	T	F	F	F

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Subset Sum Wrap

Backtracking running time $O(2^n)$

What are our subproblems?

At an arbitrary iteration $1 \le i \le n+1$ and $t \le T$

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What are the space/time requirements?

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Time: $O(nT)$

$O(nT)$

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```

Is FastSubsetSum *always* faster than the recursive version?

Subset Sum Wrap

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  return $S[1,T]$
```

Is FastSubsetSum *always* faster than the recursive version?

No! If $T \gg 2^n$, the recursive version is actually faster!

Edit Distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.

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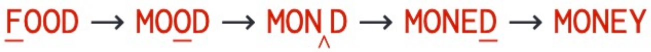

$\underline{\mathrm{F}}\mathrm{OOD} \rightarrow \mathrm{MO}\underline{\mathrm{O}}\mathrm{D} \rightarrow \mathrm{MON}_{\wedge}\mathrm{D} \rightarrow \mathrm{MONE}\underline{\mathrm{D}} \rightarrow \mathrm{MONEY}$

Edit Distance

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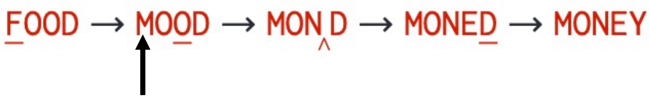
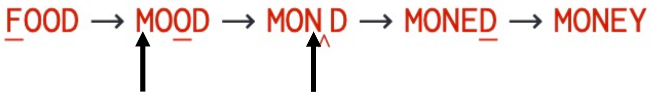


Substitution

Edit Distance

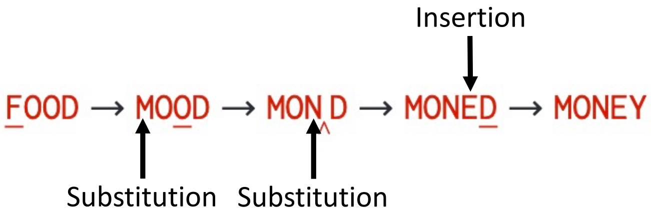
The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



Substitution Substitution

Edit Distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



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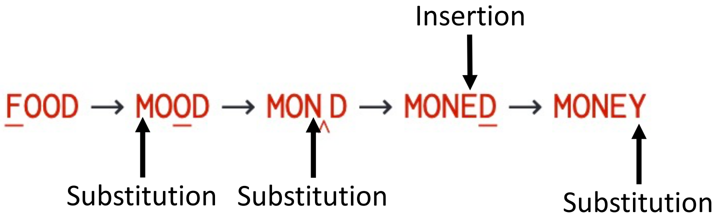

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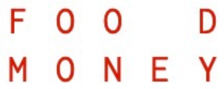
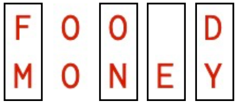


F	O	O		D
M	O	N	E	Y

Alternative: Align the strings and count the differences

Edit Distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.

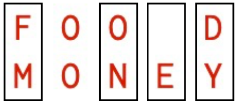


F	O	O		D
M	O	N	E	Y

Alternative: Align the strings and count the differences

Edit Distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.

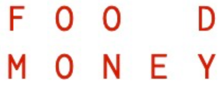


$EditDistance(\text{food}, \text{money}) = 4$

Alternative: Align the strings and count the differences

Formulating a recursive edit distance

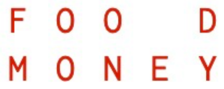
The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



```
F O O   D
M O N E Y
```

Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.

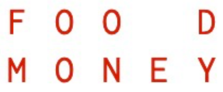


```
F O O   D
M O N E Y
```

What should our subproblems be?

Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



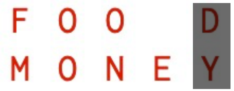
F	O	O		D
M	O	N	E	Y

What should our subproblems be?

- Imagine that we have this alignment representation for the optimal edit distance

Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



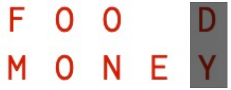
F	O	O		D
M	O	N	E	Y

What should our subproblems be?

- Imagine that we have this alignment representation for the optimal edit distance
- Remove the last column

Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



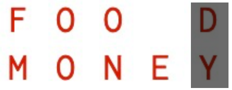
```
F O O   D
M O N E Y
```

What should our subproblems be?

- Imagine that we have this alignment representation for the optimal edit distance
- Remove the last column
- What must be true of the remaining prefixes?

Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.



F	O	O		D
M	O	N	E	Y

What should our subproblems be?

- Imagine that we have this alignment representation for the optimal edit distance
- Remove the last column
- What must be true of the remaining prefixes?
    - They must also be optimal!

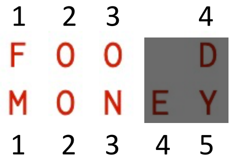
Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.

For any two input strings $A[1..n]$ and $B[1..m]$, let

$$Edit(i, j)$$

denote the edit distance between prefixes $A[1..i]$ and $B[1..j]$. We need to compute $Edit(n, m)$.



1	2	3		4
F	O	O		D
M	O	N	E	Y
1	2	3	4	5

What should our subproblems be?

- Imagine that we have this alignment representation for the optimal edit distance
- Remove the last column
- What must be true of the remaining prefixes?
  - They must also be optimal!

Formulating a recursive edit distance

The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions that will transform one string into the other.

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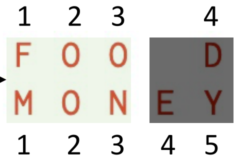
denote the edit distance between prefixes $A[1..i]$ and $B[1..j]$. We need to compute $Edit(n, m)$.

Decisions here do not depend on what was already computed!

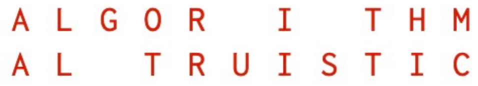


What should our subproblems be?

- Imagine that we have this alignment representation for the optimal edit distance
- Remove the last column
- What must be true of the remaining prefixes?
    - They must also be optimal!

Formulating a recursive edit distance

Each call to $Edit(i, j)$makes a decision about how to align the last column in the substring.
There are three possibilities:



```
A L G O R   I   T H M
A L   T R U I S T I C
```

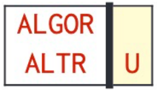
Formulating a recursive edit distance

Each call to $Edit(i,j)$makes a decision about how to align the last column in the substring.
There are three possibilities:

A	L	G	O	R		I		T	H	M
A	L		T	R	U	I	S	T	I	C

Arbitrary Case

1. Insertion



$$Edit(i,j-1)+1$$

2. Deletion

3. Substitution

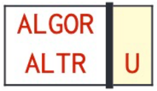
Formulating a recursive edit distance

Each call to $Edit(i, j)$makes a decision about how to align the last column in the substring.
There are three possibilities:

A	L	G	O	R		I		T	H	M
A	L		T	R	U	I	S	T	I	C

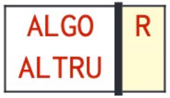
Arbitrary Case

1. Insertion


ALGOR
ALTR
U

$$Edit(i, j-1) + 1$$

2. Deletion


ALGO
ALTRU
R

$$Edit(i-1, j) + 1$$

3. Substitution

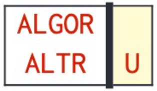
Formulating a recursive edit distance

Each call to $Edit(i, j)$makes a decision about how to align the last column in the substring.
There are three possibilities:

```
A L G O R   I   T H M
A L   T R U I S T I C
```

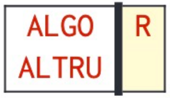
Arbitrary Case

1. Insertion



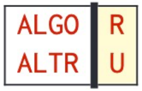
$Edit(i, j-1)+1$

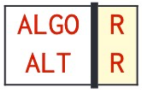
2. Deletion



$Edit(i-1, j)+1$

3. Substitution





$Edit(i-1, j-1)+1$, if $A[i] \neq B[j]$

$Edit(i-1, j-1)$, if $A[i] = B[j]$

Formulating a recursive edit distance

sear
ears

ears
sear

Each call to $Edit(i, j)$makes a decision about how to align the last column in the substring.
There are three possibilities:

A	L	G	O	R		I		T	H	M
A	L		T	R	U	I	S	T	I	C

Arbitrary Case

Base Case

1. Insertion

ALGOR	
ALTR	U

$Edit(i, j-1) + 1$

$Edit(0, j) = j$ to insert gaps in $A[0..j]$

2. Deletion

ALGO	R
ALTRU	

$Edit(i-1, j) + 1$

$Edit(i, 0) = i$ to delete characters from $B[0..i]$

3. Substitution

ALGO	R
ALTR	U

ALGO	R
ALT	R

$Edit(i-1, j-1) + 1$, if $A[i] \neq B[j]$

$Edit(i-1, j-1)$, if $A[i] = B[j]$

Formulating a recursive edit distance

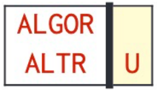
Each call to $Edit(i,j)$makes a decision about how to align the last column in the substring.
There are three possibilities:

$$Edit(i,j) = \begin{cases} i & \text{if } j = 0 \\ j & \text{if } i = 0 \\ \min \left\{ \begin{array}{c} Edit(i, j-1) + 1 \\ Edit(i-1, j) + 1 \\ Edit(i-1, j-1) + [A[i] \neq B[j]] \end{array} \right\} & \text{otherwise} \end{cases}$$

Arbitrary Case

Base Case

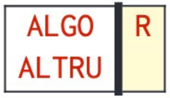
1. Insertion



$Edit(i, j-1) + 1$

$Edit(0, j) = j$ to insert gaps in $A[0..j]$

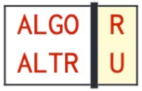
2. Deletion

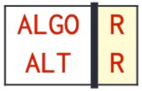


$Edit(i-1, j) + 1$

$Edit(i, 0) = i$ to delete characters from $B[0..i]$

3. Substitution





$Edit(i-1, j-1) + 1$, if $A[i] \neq B[j]$

$Edit(i-1, j-1)$, if $A[i] = B[j]$

Next Time

We will formulate a dynamic programming algorithm for Edit Distance and discuss the Knapsack Problem.

$$Edit(i,j) = \begin{cases} i & \text{if } j = 0 \\ j & \text{if } i = 0 \\ \min \left\{ \begin{array}{c} Edit(i, j-1) + 1 \\ Edit(i-1, j) + 1 \\ Edit(i-1, j-1) + [A[i] \neq B[j]] \end{array} \right\} & \text{otherwise} \end{cases}$$

Wrap up

No new reading assignment (still chapter 3 of Erickson)

- If you didn’t follow our Edit Distance discussion, read 3.7 before Monday!

Work on homework 2! Ask questions on Piazza.

Midterm next Weds 8PM – Fri 8PM.

- Topics will be everything we have done so far:
  - Asymptotic analysis and Divide and Conquer (including recursion, backtracking, and dynamic programming)
- If there are things you have struggled with, strategize sooner rather than later about how you will review them before Wednesday!

Enjoy your weekend!